On the Gravitational Interaction of Light

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Outline

- **The light-light gravitational interaction**

  First studies on the light-light interaction go back to Tolman, Ehrenfest and Podolsky 1931 and, later, to Wheeler 1955 who analysed the gravitational field of light beams and the corresponding geodesics in the linear approximation of Einstein equations. They discovered that null rays behave differently according to whether they propagate parallel or antiparallel to a steady, long, straight beam of light, but they didn't provide a physical explanation of this fact.

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They also extended the analysis to the realm of exact *pp*-wave solutions of the Einstein equations.

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- **Our point of view**

  In the usual treatment of gravitational waves only Fourier expandable solutions of d'Alembert equation are considered; then it is possible to choose a special gauge (TT-gauge) which kills the spin-0 and spin-1 components.

  However there exist (see section 2 and 3) physically meaningful solutions (Peres 1959 Stephani 1996, Stephani, Kramer, MacCallum, Honselaers and Herlt 2003, Canfora, Vilasi and Vitale 2002) of Einstein equations which are not Fourier expandable and nevertheless whose associated energy is finite.

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- For some of these solutions the standard analysis shows that spin-1 components cannot be killed (Canfora and Vilasi 2004, Canfora, Vilasi and Vitale 2004); this implies that repulsive aspects of gravity are possible within pure General Relativity. It was also shown that light is among possible sources of such spin-1 waves (Vilasi 2007).

An integrable case

Some decades ago, Belinsky and Zakharov were able to solve Einstein field equations in vacuum for a metric of the form

$$g = f(z,t)\left(dt^2 - dz^2\right) + h_{11}(z,t)\,dx^2 + h_{22}(z,t)\,dy^2 + 2h_{12}(z,t)\,dxdy.$$

Indeed, the Einstein equations essentially reduce to

$$\left(\alpha \mathbf{H}^{-1}\mathbf{H}_{\xi}\right)_{\eta} + \left(\alpha \mathbf{H}^{-1}\mathbf{H}_{\eta}\right)_{\xi} = 0,$$

$$\mathbf{H} \equiv \|h_{ab}\|, \;\; \xi = (t+z)/\sqrt{2}, \quad \eta = (t-z)/\sqrt{2}, \quad \alpha = \sqrt{|\det \mathbf{H}|}.$$

which is a non-linear partial differential equation whose *generalized Lax form* is characteristic for integrable systems. Thus, by using a suitable generalization of the *Inverse Scattering Transform*, they were able to find *solitary waves solutions* of Einstein field equations.

The function f is determined by the equations

$$\alpha,_i \partial_i (\ln |f|) = \alpha,_{ii} - \alpha,_i^2 / 2\alpha$$

i.e. by pure quadrature, in terms of α.

- **Integrable systems**

  Integrable systems generally exhibit a *recursion operator* which is responsible for the construction of a sequence of conserved functionals. It is the endomorphism associated with a (1,1) tensor field which is invariant for the dynamics, has a vanishing Nijenhuis torsion and special spectral properties which allow to generalize (De Filippo, Marmo, Vilasi 1982, 1984, 1985, 1994), to infinite dimensional manifolds, the classical Liouville theorem on complete integrability.

  How many conserved functionals do exist in our gravitational case and which is their physical significance?

- **Geometric properties?**

  A geometric look at mentioned metrics shows that they are invariant under translations along the x, y-axes, *i.e.* they admit two Killing fields, ∂_x and ∂_y, closing on an Abelian 2-dimensional Lie algebra $\mathcal{A}_2$. Moreover, the distribution $\mathcal{D}$ generated by ∂_x and ∂_y is 2−dimensional and the distribution $\mathcal{D}^\perp$ orthogonal to $\mathcal{D}$ is integrable and transversal to $\mathcal{D}$ (*i.e.* $\mathcal{D}^\perp$ and $\mathcal{D}$ have in common only the vanishing vector field).

- **Integrable distribution?**

  A 2−dimensional distribution is called *integrable* if the Faraday force lines (*integral curves* in differential geometric language) of two generating vector fields are *surfaces forming*, that is they mesh one another as cotton threads in a web. Such surfaces are called leaves of the distribution. A non integrable 2−dimensional distribution is called *semi-integrable* if it is part (*i.e.*, a suitable restriction) of a 3−dimensional integrable distribution.

Since a 2−dimensional Lie algebra is either Abelian ($\mathcal{A}_2$) or non-Abelian ($\mathcal{G}_2$), it has been natural to consider (Sparano, Vilasi, Vinogradov 2000, 2001, 2002) the problem of characterizing all gravitational fields g admitting a Lie algebra $\mathcal{G}$ of Killing fields such that:

I the distribution $\mathcal{D}$, generated by vector fields of $\mathcal{G}$, is 2-dimensional;

II the distribution $\mathcal{D}^{\perp}$, orthogonal to $\mathcal{D}$ is integrable and transversal to $\mathcal{D}$.

The condition of transversality can be relaxed (Catalano-Ferraioli, Vinogradov 2004, Baetchold 2005), so that in order to distinguish the different cases, the notation $(\mathcal{G}, r)$ is used: metrics satisfying the conditions I and II are called *of* $(\mathcal{G}, 2)-$*type*; metrics satisfying conditions I and II, except the transversality condition, are called *of* $(\mathcal{G}, 0)-$*type* or *of* $(\mathcal{G}, 1)-$*type* according to the rank r of their restriction the leaves of $\mathcal{D}$ which are also called *Killing leaves*.

All the possible situations, corresponding to a $2-$dimensional Lie algebra of isometries, are described by the following table in which the cases indicated with bold letters are essentially solved.

A 2−dimensional Lie Algebra Ricci-flat metric classification

Table:

	$\mathcal{D}^{\perp}$, $r=0$	$\mathcal{D}^{\perp}$, $r=1$	$\mathcal{D}^{\perp}$, $r=2$
$\mathcal{G}_2$	**integrable**	**integrable (1K)**	**integrable**
$\mathcal{G}_2$	**semi-integrable**	**semi-integrable**	**semi-integrable**
$\mathcal{G}_2$	**NOn-integrable**	**non-integrable (1K)**	non-integrable?
$\mathcal{A}_2$	**integrable**	**integrable (3)**	**integrable (BZ)**
$\mathcal{A}_2$	semi-integrable?	semi-integrable?	**semi-integrable**
$\mathcal{A}_2$	**NOn-integrable**	**NOn-integrable**	non-integrable?

The study of $\mathcal{A}_2$-invariant Einstein metrics goes back to Einstein and Rosen (1937), Rosen (1954), Kompaneyets (1958), Geroch (1972) , Belinsky, Khalatnikov, Zakharov (1978), so that some exact solutions already known in the literature have been rediscovered. Nevertheless, our geometric approach allows to perform in a natural way the choice of coordinates, i.e., the coordinates adapted to the symmetries of the metrics, even if they do not admit integrable $\mathcal{D}^{\perp}$ distribution. Usually, the standard techniques to find exact solutions assume, from the very beginning, that there exist natural vector fields, surfaces forming, which simplify the choice of the coordinates system. These assumptions (besides the Lie algebra of Killing fields) are strong topological constraints on the spacetime: the present approach can be applied also when such topological assumptions do not hold.

Outline

Geometric properties

Over the past years (2001 - 2011) a family of exact solutions g of Einstein field equations, some of them representing gravitational waves generated by a beam of light, has been explicitly written (Sparano, Vilasi, Vinogradov, Canfora, Vitale)

$$g = 2f(dx^2 + dy^2) + \mu\left[(w(x,y) - 2q)dp^2 + 2dpdq\right], \tag{1}$$

where $\mu(x,y) = A\Phi(x,y) + B$ (with $\Phi(x,y)$ a harmonic function and A, B numerical constants), $f(x,y) = (\nabla\Phi)^2\sqrt{|\mu|}/\mu$, and $w(x,y)$ is solution of the *Euler-Darboux-Poisson equation:*

$$\Delta w + (\partial_x \ln|\mu|)\,\partial_x w + (\partial_y \ln|\mu|)\,\partial_y w = \rho,$$

$T_{\mu\nu} = \rho\delta_{\mu 3}\delta_{\nu 3}$ representing the energy-momentum tensor and Δ the Laplace operator in the (x,y) −plane.

Previous metrics are invariant for the non Abelian Lie agebra $\mathcal{G}_2$ of Killing fields

$$X = \frac{\partial}{\partial p}, \qquad Y = e^p \frac{\partial}{\partial q},$$

with

$$[X, Y] = Y, \qquad g(Y, Y) = 0,$$

generating a 2−dimensional distribution $\mathcal{D}$ whose *orthogonal distribution* $\mathcal{D}^{\perp}$ is integrable.

In the particular case $s = 1$, $f = 1/2$ and $\mu = 1$, the above family is locally diffeomorphic to a subclass of Peres solutions and, by using the transformation

$$p = \ln|u| \qquad q = uv,$$

can be written in the form

$$g = dx^2 + dy^2 + 2dudv + \frac{w}{u^2}du^2, \tag{2}$$

with $\Delta w(x, y) = \rho$, and has the Lorentz invariant *Kerr-Schild* form:

$$g_{\mu\nu} = \eta_{\mu\nu} + V k_\mu k_\nu, \qquad k_\mu k^\mu = 0.$$

- **Wave Character**
  The wave character and the polarization of these gravitational fields has been analyzed in many ways. For example, the Zel'manov criterion (Zakharov 1973) was used to show that these are gravitational waves and the propagation direction was determined by using the Landau-Lifshitz pseudo-tensor. However, the algebraic Pirani criterion is easier to handle since it determines both the wave character of the solutions and the propagation direction at once. Moreover, it has been shown that, in the vacuum case, the two methods agree. To use this criterion, the Weyl scalars must be evaluated according to Petrov classification (Petrov 1955, Petrov 1969).

In the Newmann-Penrose formulation (Penrose 60, Neumann and Penrose 62) of Petrov classification, we need a *tetrad* basis with two real null vector fields and two real spacelike (or two complex null) vector fields. Then, if the metric belongs to type **N** of the Petrov classification, it is a gravitational wave propagating along one of the two real null vector fields (Pirani criterion). Let us observe that ∂_x and ∂_y are spacelike real vector fields and ∂_v is a null real vector but ∂_u is not. With the transformation $x \mapsto x, \quad y \mapsto y, \quad u \mapsto u, \quad v \mapsto v + w(x,y)/2u$, whose Jacobian is equal to one, the metric (2) becomes:

$$g = dx^2 + dy^2 + 2dudv + dw(x,y)dln|u|. \tag{3}$$

Since ∂_x and ∂_y are spacelike real vector fields and ∂_u and ∂_v are null real vector fields, the above set of coordinates is the right one to apply for the Pirani's criterion.

Since the only nonvanishing components of the Riemann tensor, corresponding to the metric (3), are

$$R_{iuju} = \frac{2}{u^3}\partial^2_{ij} w(x,y), \qquad i,j = x,y$$

these gravitational fields belong to Petrov type **N** (Zakharov 73). Then, according to the Pirani's criterion, previous metric does indeed represent a gravitational wave propagating along the null vector field ∂_u.

It is well known that linearized gravitational waves can be characterized entirely in terms of the linearized and gauge invariant Weyl scalars. The non vanishing Weyl scalar of a typical spin-2 gravitational wave is Ψ_4. Metrics (3) also have as non vanishing Weyl scalar Ψ_4.

- **Spin**
  Besides being an exact solution of Einstein equations, the metric (3) is, for $w/u^2 << 1$, also a solution of linearized Einstein equations, thus representing a perturbation of Minkowski metric $\eta = dx^2 + dy^2 + 2dudv = dx^2 + dy^2 + dz^2 - dt^2$ (with $u = (z-t)/\sqrt{2} \quad v = (z+t)/\sqrt{2}$) with the perturbation, generated by a light beam or by a photon wave packet moving along the z-axis, given by

$$h = dw(x,y)dln|z-t|,$$

whose non vanishing components are

$$h_{0,1} = -h_{13} = -\frac{w_x}{(z-t)} \quad h_{0,2} = -h_{23} = -\frac{w_y}{(z-t)}$$

- **The energy-momentum tensor**

  A transparent method to determine the spin of a gravitational wave is to look at its physical degrees of freedom, *i.e.* the components which contribute to the energy (Dirac 75). One should use the Landau-Lifshitz (pseudo)-tensor t^{μ}_{ν} which, in the asymptotically flat case, agrees with the Bondi flux at infinity (Canfora, Vilasi and Vitale 2004). It is worth to remark that the canonical and the Landau-Lifchitz energy-momentum pseudo-tensors are true tensors for Lorentz transformations. Thus, any Lorentz transformation will preserve the form of these tensors; this allows to perform the analysis according to the Dirac procedure. A globally square integrable solution $h_{\mu\nu}$ of the wave equation is a function of $r = k_{\mu}x^{\mu}$ with $k_{\mu}k^{\mu} = 0$.

- With the choice $k_\mu = (1, 0, 0, -1)$, we get for the energy density t_0^0 and the energy momentum t_0^3 the following result:

$$16\pi t_0^0 = \frac{1}{4}\left(u_{11} - u_{22}\right)^2 + u_{12}^2, \qquad t_0^0 = t_0^3$$

where $u_{\mu\nu} \equiv dh_{\mu\nu}/dr$. Thus, the physical components which contribute to the energy density are $h_{11} - h_{22}$ and h_{12}. Following the analysis of Dirac 1975, we see that they are eigenvectors of the infinitesimal rotation generator $\mathcal{R}$, in the plane $x - y$, belonging to the eigenvalues $\pm 2i$. The components of $h_{\mu\nu}$ which contribute to the energy thus correspond to spin-2.

- In the case of the prototype of spin-1 gravitational waves (3), both Landau-Lifchitz energy-momentum pseudo-tensor and Bel-Robinson tensor (Bel 1958, Robinson 1958; Bel 1959) single out the same wave components and we have:

$$\tau_0^0 \sim c_1 (h_{0x,x})^2 + c_2 (h_{0y,x})^2, \quad t_0^0 = t_0^3$$

where c_1 e c_2 constants, so that the physical components of the metric are h_{0x} and h_{0y}. Following the previous analysis one can see that these two components are eigenvectors of $i\mathcal{R}$ belonging to the eigenvalues ± 1. In other words, metrics (3), which are not pure gauge since the Riemann tensor is not vanishing, represent spin-1 gravitational waves propagating along the $z-$axis at light velocity.

- **Summarizing**
  Globally square integrable spin-1 gravitational waves propagating on a flat background are always pure gauge.
- *Spin-1 gravitational waves which are not globally square integrable are not pure gauge.* It is always possible to write metric (3) in an apparently transverse gauge (Stefani 96); however since these coordinates are no more harmonic this transformation is not compatible with the linearization procedure.
- What truly distinguishes spin-1 from spin-2 gravitational waves is the fact that in the spin-1 case the Weyl scalar has a non trivial dependence on the transverse coordinates (x, y) due to the presence of the harmonic function. This could led to observable effects on length scales larger than the *characteristic length scale* where the harmonic function changes significantly.

- Indeed, the Weyl scalar enters in the geodesic deviation equation implying a non standard deformation of a ring of test particles breaking the invariance under of π rotation around the propagation direction. Eventually, one can say that there should be distinguishable effects of spin-1 waves at suitably large length scales.
- It is also worth to stress that the results of Aichelburg and Sexl 1971, Felber 2008 and 2010, van Holten 2008 suggest that the sources of asymptotically flat $pp-$waves (which have been interpreted as spin-1 gravitational waves Canfora, Vilasi and Vitale 2002 and 2004) repel each other. Thus, in a field theoretical perspective (see Appendix), $pp-$gravitons" must have spin-1 .

- **An alternative description**

Hereafter the spatial part of four-vectors will be denoted in bold and the standard symbols of three-dimensional vector calculus will be adopted. Metric (3) can be written in the GravitoElectroMagnetic form (GEM)

$$g = (2\Phi^{(g)} - 1)dt^2 - 4(\mathbf{A}^{(g)} \cdot d\mathbf{r})dt + (2\Phi^{(g)} + 1)d\mathbf{r} \cdot d\mathbf{r}, \qquad (4)$$

where

$$\mathbf{r} = (x, y, z)\,, \quad 2\Phi^{(g)} = h_{00}, \quad 2A_i^{(g)} = -h_{0i}.$$

- **Gravito-Lorentz gauge**

In terms of $\Phi^{(g)}$ and $\mathbf{A}^{(g)}$ the harmonic gauge condition reads

$$\frac{\partial \Phi^{(g)}}{\partial t} + \frac{1}{2} \nabla \cdot \mathbf{A}^{(g)} = 0, \tag{5}$$

and, once the gravitoelectric and gravitomagnetic fields are defined in terms of g-potentials, as

$$\mathbf{E}^{(\mathbf{g})} = -\nabla \Phi^{(g)} - \frac{1}{2} \frac{\partial \mathbf{A}^{(g)}}{\partial t}, \qquad \mathbf{B}^{(\mathbf{g})} = \nabla \wedge \mathbf{A}^{(g)},$$

one finds that the Einstein equations resemble Maxwell equations. Consequently, being the dynamics fully encoded in Maxwell-like equations, this formalism describes the physical effects of the vector part of the gravitational field.

- **Gravito-Faraday tensor**

  Gravitational waves can be also described in analogy with electromagnetic waves, the gravitoelectric and the gravitomagnetic components of the metric being

$$E^{(g)}_{\mu} = F^{(g)}_{\mu 0}; \qquad B^{(g)\mu} = -\varepsilon^{\mu 0 \alpha\beta} F^{(g)}_{\alpha\beta}/2 \quad ,$$

  where

$$\begin{aligned} F^{(g)}_{\mu\nu} &= \partial_{\mu} A^{(g)}_{\nu} - \partial_{\nu} A^{(g)}_{\mu} \\ A^{(g)}_{\mu} &= -h_{0\mu}/2 = (-\Phi^{(g)}, \mathbf{A}^{(g)}). \end{aligned}$$

Outline

- **Geodesic motion**

  The geodesic motion of a *massive particle* moving with velocity $v^{\mu} = (1, \underline{\mathrm{v}}), |\mathbf{v}| << \mathbf{1}$, in a light beam gravitational field characterized by gravitoelectric $\mathbf{E}^{(g)}$ and gravitomagnetic $\mathbf{B}^{(g)}$ fields, is determined (at first order in $|\mathbf{v}|$) by the *acceleration*:

$$\mathbf{a}^{(g)} = -\mathbf{E}^{(g)} - 2\mathbf{v} \wedge \mathbf{B}^{(g)}.$$

- The geodesic motion of a *massless particle* moving with velocity $v^{\mu} = (1, \mathbf{v}), |\mathbf{v}| = 1$, in the light beam gravitational field, parallel(anti) to z-axis $(v_j = \pm\delta_{j3})$ is slightly different

$$\mathbf{a}^{(g)} = -2\left(\mathbf{E}^{(g)} + \mathbf{v} \wedge \mathbf{B}^{(g)}\right).$$

  There are two contributions, one by the light beam, which is the source of gravity, and the other by the test photon.

The gravitoelectric and gravitomagnetic fields corresponding to our metric are given by

$$\mathbf{E}^{(g)} = (w_x, w_y, 0)/4u^2, \quad \mathbf{B}^{(g)} = (w_y, -w_x, 0)/4u^2,$$

and the "gravitational acceleration" of a massless particle will be

$$\mathbf{a}^{(g)} = -[w_x(1-v_z)\mathbf{i} + w_y(1-v_z)\mathbf{j} + (w_x v_x + w_y v_y)\mathbf{k}]/2u^2.$$

Notice that
$\mathbf{E}^{(g)} \cdot \mathbf{B}^{(g)} = 0 \qquad \left|\mathbf{E}^{(g)}\right|^2 - \left|\mathbf{B}^{(g)}\right|^2 = 0$

The photon velocity

The velocity **v** of a photon is determined by the null geodesics equations

$$(h-1) - 2hv_z + (h+1)v_z^2 = 0$$

which has two solutions

$$v_z = 1, \qquad v_z = \frac{h-1}{h+1} = \frac{w-u^2}{w+u^2}$$

.

If the photon propagates parallel to the light beam, $v = (0, 0, 1)$, then

$$\mathbf{a}^{(g)} = 0$$

and there is not attraction or repulsion (see also Zee 2003).
If the photon does not propagate parallel to the light beam, the velocity will be $v = (h - 1) / (h + 1)$, then

$$\mathbf{a}^{(g)} = -\nabla w / 2 \left(w + u^2\right)$$

and the force turns out to be attractive.

- Thus, the lack of attraction found by Tolman, Ehrenfest, Podolsky (later also analysed by Wheeler, Faraoni and Dumse) comes out also from the analysis of the geodesical motion of a massless spin-1 test particle in the strong gravitational field of the light, neglecting however the gravitational field generated by that particle. An exhaustive answer could derive only determining the gravitational field generated by two photons, each one generating spin-1 gravitational waves. However, since helicity seems to play for photons the same role that charge plays for charged particles, two photons with the same helicity should repel one another. This repulsion turns out to be very weak and cannot be certainly observed in laboratory but it could play a relevant role at cosmic scale and could give not trivial contributions to the dark energy. Thus, together with gravitons (spin-2), one may postulate the existence of graviphotons (spin-1) and graviscalar (spin-0).

Outline

- Relativistic jets are extremely powerful jets of plasma which emerge from presumed massive objects at the centers of some active radio galaxies and quasars. Their lengths can reach several thousand or even hundreds of thousands of light years. Among the different types of astrophysical jets, the most energetic ones are potential candidates to give rise to emission of gravitational waves. For example, highly relativistic jets should be associated with some sources of gamma ray bursts (GRBs) (Piran T 2004, Sari R, Piran T and Halpern J P 1999, Piran T 2000, Meszaros P 1999). The impact of an ultra relativistic jet over the space-time metric can be studied starting from the extreme situation where the velocity of the particles in the beam is assumed to be equal to the velocity of light. The jet is then represented by a beam of null particles.

- The relativistic interaction of a photon with the jet can be described by the geodesic motion in the light gravitational field. For a flow of radiation of a null electromagnetic (*em*) field along the z-axis, the (*em*) energy-momentum tensor macroscopic components $T_{\mu\nu} = F_{\mu\alpha}F^{\alpha}_{\nu} + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$ reduce to

$$T_{00} = \frac{\rho}{z - ct}, \quad T_{03} = T_{30} = -\frac{\rho}{z - ct}, \quad T_{33} = \frac{\rho}{z - ct}$$

where $\rho = \left(E^2 + B^2\right)/2$ represents the amplitude of the field, $i.e.$ the density of radiant energy at point of interest. They are just the components in the coordinates (t, x, y, z) of the energy-momentum tensor $T = \rho du^2$ of section 17.

We assume then that the energy density is a constant ρ_0 within a certain radius $0 \leq r = \sqrt{x^2 + y^2} \leq r_0$ and vanishes outside. Thus, the source represents a cylindrical beam with width r_0 and constitutes a simple generalization of a single null particle.
Introducing back the standard coupling constant of Einstein tensor with matter energy-momentum tensor, we have:

$$\Delta w(x, y) = \frac{8\pi G}{c^4}\rho. \tag{6}$$

- The cylindrical symmetry implies that $w(x,y)$ will depend only on the distance r from the beam. A solution $w(r)$ of Poisson equation (6) satisfying the continuity condition at $r = r_0$ can be easily written as

$$w(r) = \frac{4\pi G}{c^4}\rho_0 r^2 \quad r \leq r_0 \tag{7}$$

$$w(r) = \frac{8\pi G}{c^4}\rho_0 r_0^2 \left[\ln\left(\frac{r}{r_0}\right) + \frac{1}{2}\right] \qquad r > r_0 \tag{8}$$

Or also

$$w(r) = \frac{4\pi G\rho_0}{c^4} r_0^2 W\left(r\right) \tag{9}$$

with

$$W\left(r\right) = \begin{cases} r^2/r_0^2 & r < r_o \\ 1 + \ln\left(\frac{r}{r_0}\right)^2 & r > r_0 \end{cases} \tag{10}$$

- so that a photon moving antiparallel and external to the beam will experience at the space-time point (t, x, y, z) a transversal gravitational attraction espressed by

$$\mathbf{a}^{(g)}(t, x, y, z) = -\frac{16\pi G}{c^4}\rho_0 r_0^2 \frac{\mathbf{r}}{r^2 (z - ct)^2}, \tag{11}$$

  where the speed of light c has been reintroduced and the retardation is automatically accounted for. As a consequence of spin-1 of our wave and of QFT a photon moving parallel and external to the beam will experience at the space-time point (t, x, y, z) a transversal gravitational repulsion given by

$$\mathbf{a}^{(g)}(t, x, y, z) = \frac{16\pi G}{c^4}\rho_0 r_0^2 \frac{\mathbf{r}}{r^2 (z - ct)^2}. \tag{12}$$

- For jets which start with a small opening angle $\theta_0 \leq 10^{-3} - 10^{-4}$ (Piran T 2004, Sari R, Piran T and Halpern J P 1999, Piran T 2000, Mészáros P 1999), it can be assumed that the width of the beam remains constant during the first stage of the jet expansion (Neto, E C de Rey, de Araujo J C N, Aguiar O D, 2003) and, for a beam-length $L = c\tau \backsim 10^6 - 10^7 Km$ (a typical jet lasts $\tau \backsim 10 - 100s$), will be of the order of $r_0 = L\theta_0 \backsim 10^2 - 10^3 Km$. The energy is of the order of $E =\backsim 10^{44} - 10^{45} J$, so that $\rho_0 = E/L \backsim 10^{37} - 10^{39} J/Km$. Replacing these values in Eq. (12) and taking $G/c^4 \backsim 10^{-44} N^{-1}$, we obtain for the transversal acceleration per unit length

$$a^{(g)}(t, x, y, z) == \frac{10^{-5}}{r^2 (z - ct)^2} \quad cm^{-1},$$

where $r = \sqrt{x^2 + y^2}$ and z are the distances, expressed in cm, between the source and the point of interest and t the observation time.

- Repeating the above calculations for a laser beam in an interferometer of LIGO or VIRGO type, in the formula above we would get a factor of 10^{-50} instead of 10^{-5}. Then, the repulsion (as well as the attraction) turns out to be very weak. However it could play a relevant role at cosmic scale and could give not trivial contributions to the dark energy. At this point, together with gravitons (spin-2), one could postulate the existence of graviphotons (spin-1) and of graviscalar (spin-0) too. Through coupling to fermions, they might give forces depending on the barion number. These fields might give (Stacey F, Tuck G and Moore G 1987) two (or more) Yukawa type terms of different signs, corresponding to repulsive graviphoton exchange and attractive graviscalar exchange (range $>> 200m$). However, much more work must be done for a better understand of the role played by the gravitational field of the electromagnetic radiation and/or of null particles beams in the evolution of the universe.

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